

The radiation energy component of the Hubble function and a Λ CDM cosmological simulation

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ABSTRACT

We study some effects the inclusion of the radiation energy component in the universe, Ω_r , can have on several quantities of interest for the large-scale structure of the universe in a Λ CDM cosmological simulation; started at a very high redshift ($z = 500$). In particular we compute the power spectrum density, the halo mass function, and the concentration-mass relation for haloes. We find that Ω_r has an important contribution in the long-term nonlinear evolution of structures in the universe. For instance, a lower matter density power, by $\approx 50\%$, in all scales is obtained when compared with a simulation without the radiation term. Also, haloes formed with the Ω_r taken into account are $\approx 20\%$ less concentrated than when not included in the Hubble function.

Key words: methods: numerical, N -body simulations –cosmology: theory

1 INTRODUCTION

In the current cosmological paradigm, structures form in the Universe by amplification of primordial fluctuations driven by a gravitational instability in the expanding universe (e.g. Peacock 1999, Weinberg 2008, Mo, van den Bosh and White 2010). The growth of the instability can be studied analytically and to some extent an exploration of the weak non-linear regime may be done. However, the full non-linear evolution is essentially studied by means of numerical simulations.

The formation of non-linear structures in the universe has been studied by means of cosmological simulations, since some of the first works of, for example, Mellot et al. (1983) and Davis et al. (1985). Recent simulations have reached a high degree of complexity, both those including only dark matter (e.g. Springel et al. 2005, Diemand et al. 2008, Boylan-Kolchin et al. 2009, Heitmann et al. 2010, Klypin et al. 2011, Prada et al. 2012) and those with baryonic physics included (e.g. Wise et al. 2012, Bird et al. 2013).

Cosmological simulations considering physics outside the standard Λ CDM cosmology, such as including warm dark matter (e.g. Colín et al. 2000) and different kinds of equations for the dark matter and energy (e.g. Klypin et al. 2003, Linder & Jenkins 2003, Dolag et al. 2004, Grossi & Springel 2009, Rocha et al. 2012) have also been increasing in complexity and in the physics explored to understand the universe. Cosmological simulations with modified dynamics have also been done (e.g. Angus & Diaferio 2011).

In all of the simulations up to now, to our knowledge,

there has been no consideration of the radiation energy density contribution to the equation of motion of particles in a cosmological setting. This is in part understandable since the current cosmic background temperature of photons is $T_{\gamma 0} = 2.725$ K (e.g. Weinberg 2008), leading to an energy density of $\rho_{\gamma 0} = a_B T_{\gamma 0}^4 = 4.64 \times 10^{-34} \text{ g cm}^{-3}$, and a density parameter of photons $\Omega_\gamma = \rho_{\gamma 0} / \rho_{0c} = 2.47 \times 10^{-5} h^{-2}$; a value much smaller than the current matter density parameter $\Omega_m \approx 0.3$ and the vacuum energy density parameter $\Omega_\Lambda \approx 0.7$. Even if we consider that an additional contribution to the radiation component of the universe comes from the neutrinos from the era of $e^- + e^+$ pair annihilation the situation does not change by much; the total density of radiation (assuming massless neutrinos) becomes $\rho_{r0} = [1 + 3(7/8)(4/11)^{4/3}] \rho_{\gamma 0} = 7.80 \times 10^{-34} \text{ g cm}^{-3}$ leading to a radiation parameter $\Omega_r = \rho_{r0} / \rho_{0c} = 4.15 \times 10^{-5} h^{-2}$. Nonetheless, the effect of the radiation energy density becomes more important towards higher redshifts.

There are several problems, such as the mass function at high redshifts (e.g. Reed et al. 2007, Lukić et al. 2007), that demand a treatment as accurate as possible of the evolution of structures in the universe. For example, Reed et al. start some of their simulations at $z \approx 300$, and Lukić et al. run simulations going to as high as $z = 500$ in their study. At those starting redshifts for the simulations the radiation energy density is not negligible, due to its $(1+z)^4$ dependence. Also, due to the nonlinear way matter clusters, the effect of changing at high redshift the rate of expansion of the universe by including a radiation energy term can be significant in structures we see today.

In this *Letter* we present results of two cosmological simulations done within the standard cosmological scenario,

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but one including the radiation energy density term in the equation of motion of dark matter particles. We quantify differences between both cases in regard to the matter power spectrum, the mass function and the concentration-mass relation for halos. Other properties of the clustering of dark matter or haloes themselves are not considered here, nor a detailed study of each part is considered; such work is postponed for future communications. The objective is to point out the need to include the radiation energy term in cosmological simulations, specially those starting at high redshift, for better *consistency* with the theoretical framework of standard cosmology.

The outline of this work is as follows. In § 2 we describe the model used and describe some numerical matters. In § 3 we show some of the results of both of our simulations. Finally, in § 4 we provide a summary and final comments on this work.

2 MODELS AND NUMERICAL METHODS

In cosmological simulations the expansion of the universe has to be considered (e.g. Hockney & Eastwood 1981). In an N -body simulation with periodic boundary conditions the equation of motion of particle i is (e.g. Bertschinger 1998, Springel et al. 2001):

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i = -\frac{G}{a^3} \sum_{j \neq i} \frac{m_j(\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^3}, \quad (1)$$

where the summation goes over all periodic images of the particles j , and a is the scale factor of the Universe. The evolution of the latter follows from Friedman equation,

$$H(a) = H_0 \left(\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda \right)^{1/2}, \quad (2)$$

with $H = \dot{a}/a$, Ω_m the current epoch matter energy density parameter, Ω_r the present day radiation energy density parameter, and Ω_Λ the vacuum energy density parameter. A flat universe has been assumed in the preceding equations. Solving the coupled set of equations (1) and (2), along the corresponding Poisson's equation, determines the dynamics of the N -body simulation of the Universe.

We performed two cosmological simulations using the publicly available parallel Tree-code GADGET2 (Springel 2005). This code uses the Hubble function (2), including a curvature term, but does not include the radiation component. In GADGET2 such function is required for computing the time steps in the advancement of the motion of particles, thru drift and kick factors. We essentially modified subprograms `driftfac.c` and `timestep.c` of the GADGET2 code in order for the simulation to account for the Ω_r contribution. We will denote by Λ CDM, as is customary, the standard cosmological simulation with $\Omega_r = 0$, and with Λ rCDM the one including the radiation term in the Hubble function.

Our two simulations take as cosmological parameters those of the mean values of the WMAP7 results (Komatsu et al. 2011), where the matter density $\Omega_m = 0.275$, spectral index $n_s = 0.968$, mass fluctuation $\sigma_8 = 0.816$ and the Hubble parameter $h = 0.702$, and we take the vacuum parameter as $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$. The value of Ω_r used is that indicated in Section §1.

Initial conditions were generated, using a 2nd-order Lagrangian perturbation code (Croce, Pueblas & Scoccimarro 2006), at a redshift of $z=500$. The initial linear power spectrum density is calculated using the transfer function from the cosmic microwave background code CAMB (Lewis, Challinor & Lasenby 2000), normalized to the above σ_8 value at $z=0$. The spectrum is evolved back in time to $z=500$, using the linear growth factor D_+ (3) given by (e.g. Carroll et al. 1992, Mo et al. 2010),

$$D_+ = \frac{5\Omega_m}{2} \frac{H(z)}{H_0} \int_z^\infty \frac{(1+z)}{[H(z)/H_0]^3} dz, \quad (3)$$

to generate our initial conditions. Computing (3) at $z=500$ for the Λ CDM case gives 1.93×10^{-3} while for the Λ rCDM one 1.53×10^{-3} ; the radiation energy reduces the growth factor by $\approx 20\%$ at that redshift. Such difference will be reflected in the displacements and peculiar velocities of particles at the initial condition. This approach to modifying the initial conditions can only be considered approximate, but serves our purpose of elucidating differences when including or not the radiation term in (1).

Each simulation box has a comoving length of $L = 100 h^{-1} \text{Mpc}$ with $N_p = 512^3$ dark matter particles, leading to each particle having a mass of $m_p = 5.5 \times 10^8 h^{-1} \text{M}_\odot$. The smallest halo we are able to resolve with some confidence has a mass of $M = 100m_p \approx 6 \times 10^{10} h^{-1} \text{M}_\odot$. The gravitational (Plummer equivalent) softening length was kept at the fixed value of $\epsilon = 5 h^{-1} \text{kpc}$ in comoving coordinates. Halos were identified with the AHF public code (Gill, Knebe & Gibson 2004 and Knollmann & Knebe 2009).

3 RESULTS

Qualitative structure. The large scale structure in our box appears rather similar for both kind of simulations, however important differences appear at smaller scales. In Figure 1 we show snapshots at different times ($z=2, 1$ and $z=0$, from *left to right*) of the distribution of dark matter particles around the most massive halo ($M = 4.5 \times 10^{14} h^{-1} \text{M}_\odot$) in our Λ CDM cosmology (*top*) and the same region for that of the Λ rCDM (the halo has $M = 3.5 \times 10^{14} h^{-1} \text{M}_\odot$). At $z=0$ we find for the Λ CDM cosmology a total of 47,728 halos, while for the Λ rCDM a total of 46,473 is found.

Power spectrum. In Figure 3 we show the power spectrum density $P(k)$ computed at different redshifts from $z=5$ to $z=0$, for both kinds of cosmological evolution considered in this work. It is readily noticeable that more power density $\gtrsim 50\%$, in more than 2 decades in k , is deposited by the Λ CDM model than the Λ rCDM. The difference tends to increase at higher redshifts as shown in the bottom of Figure 3. It is worth noticing that the minimum discrepancy in power density is $\approx 50\%$ even at the smallest scale of our simulations. Not including the effect of D_+ in the initial conditions, just the Ω_r in the Hubble function, leads to a discrepancy of $\approx 15\%$ at $z=0$.

Mass function. The effect of the Ω_r term in (1) can also be seen in the halo mass function, $F(M) = N_h/V \Delta \log M$ with N_h the number of halos in a log-bin of mass M and volume V ; we computed $F(M)$ as in Lukić et al. (2007). The mass function of our haloes for different redshifts is shown

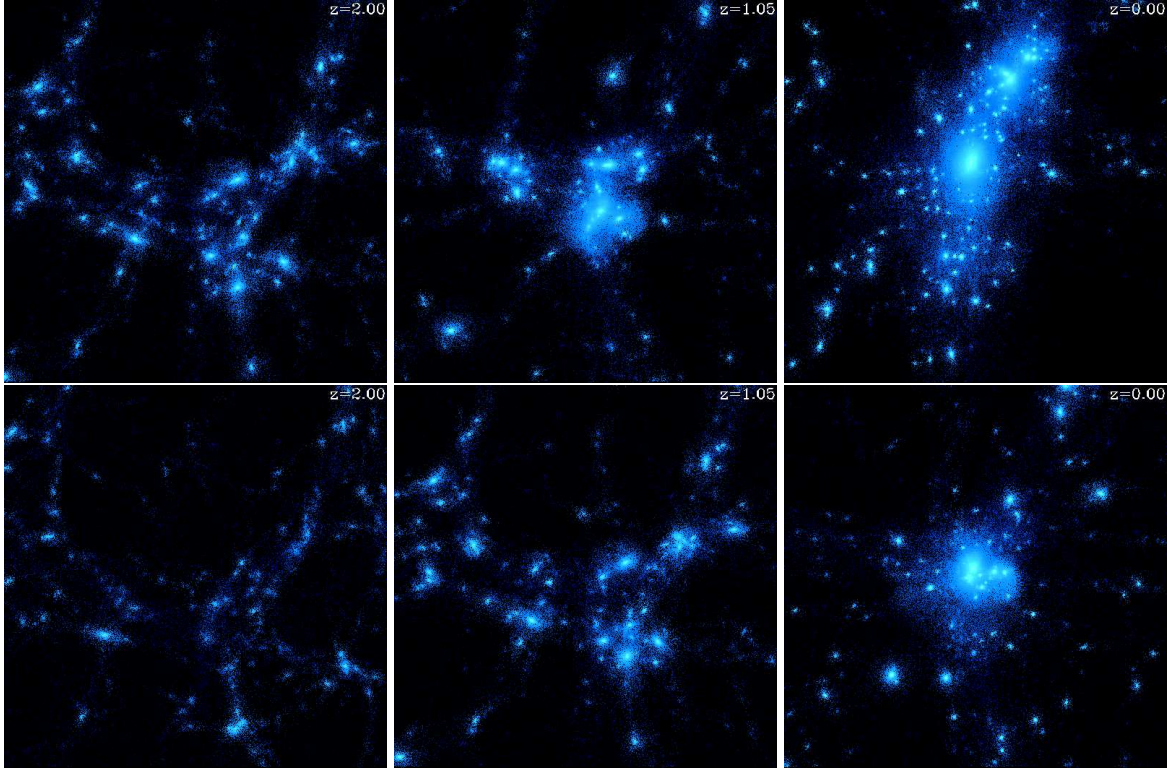


Figure 1. Projected mass distribution of particles at $z = 2, 1$ and $z = 0$, from *left to right*, in a slice of a Λ CDM (*top*) and ArCDM (*bottom*) cosmology. The length of each slice is $10 h^{-1}$ Mpc. Points are colored according to local number density. Differences after including the Ω_r term are noticeable in the snapshots; for instance, less substructures appear to be present in the ArCDM case.

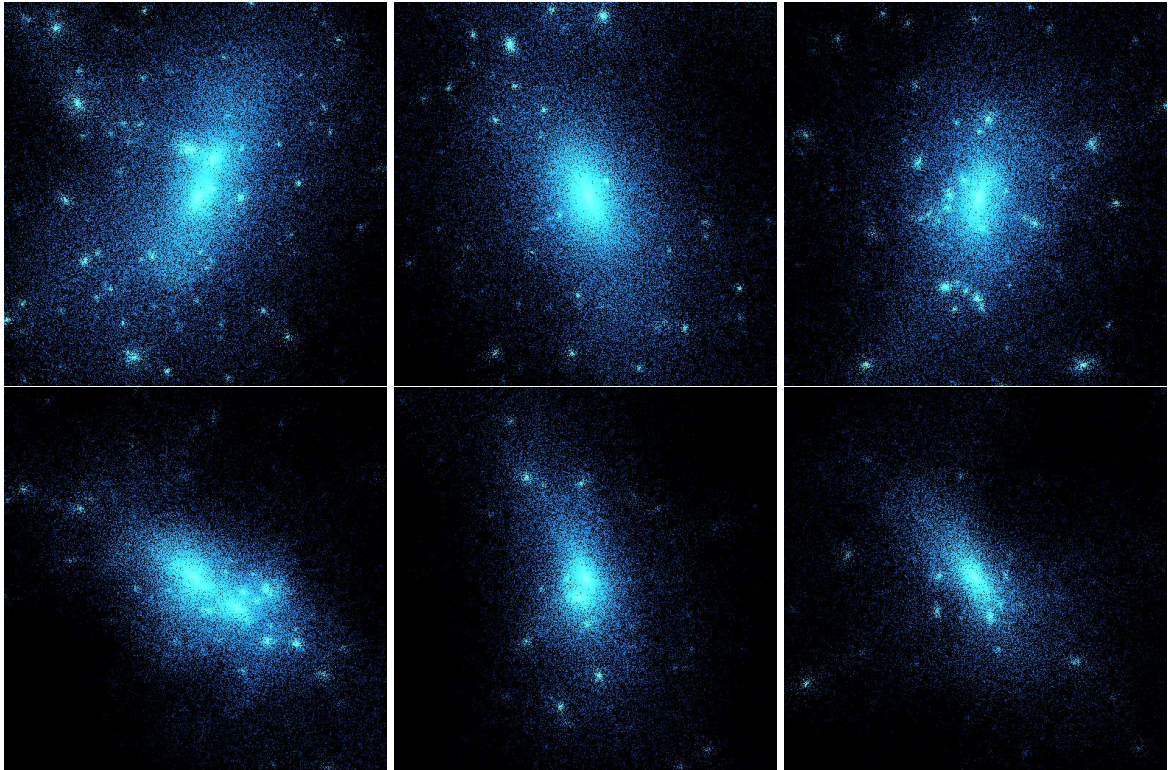


Figure 2. Projected distribution of particles of the three most massive halos in the Λ CDM cosmology (*top*) at $z = 0$, and the corresponding haloes in the ArCDM one (*bottom*). The mass of the halos decreases from *left to right*. Box size is $3 h^{-1}$ Mpc on each side. Shown are only 10 per cent of the total number of particles, and colored according to their local density as in Figure 1.

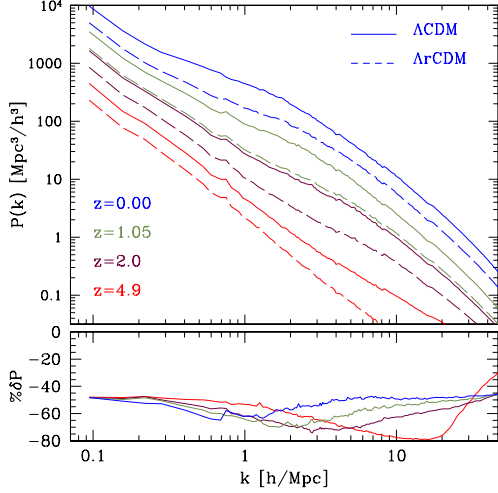


Figure 3. Power spectrum of the simulation with (dashed lines) and without (solid lines) the Ω_r term. At the bottom the fractional change, in percent, $\delta P = (P_{\Lambda\text{CDM}} - P_{\text{ArCDM}})/P_{\Lambda\text{CDM}}$, is shown. Not including Ω_r leads to a higher power density at all scales; at least to the resolution limit of this study.

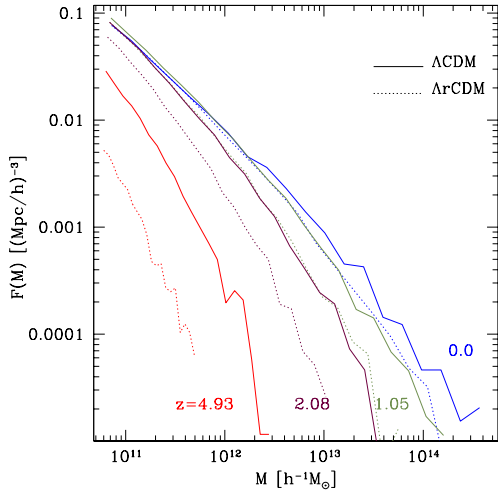


Figure 4. Mass function, $F(M)$, for halos in a ΛCDM (solid lines) and ArCDM (dashed lines) cosmology. Redshifts corresponding to different epochs are indicated in the plot. A lower number of halos at higher redshifts are formed in an ArCDM cosmology.

in Figure 4. As noted, the mass function at $z=0$ is somewhat similar with and without the radiation energy density term, but the effect of the latter is stronger toward higher redshifts; as was also indicated by the behaviour of the $P(k)$. Including the Ω_r term leads to a lowering of the formation of halos at higher redshifts. This can have important consequences for the demographics of haloes that form galaxies, and one may speculate that at much higher redshifts in the abundance of primordial haloes that would host the first stars in the Universe.

Haloes concentration-mass relation. As a preliminary result on the properties of halos formed under the two cosmologies explored here, we computed the mean concentration-mass relation, $c(M)$, for the halos found in our simulations. Haloes are assumed to follow a NFW pro-

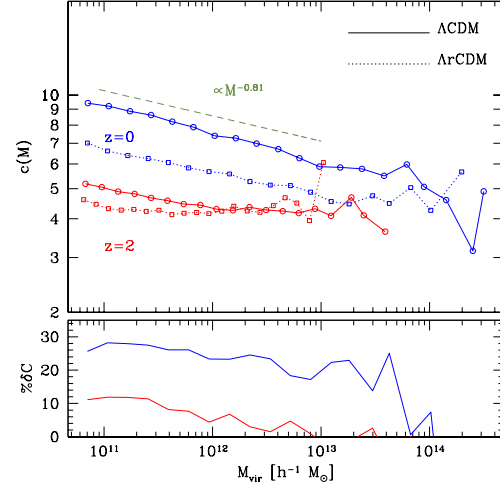


Figure 5. Mean concentration-mass relation, $c(M)$, for halos found in both of our cosmological runs. Halos in the ArCDM (solid lines) cosmology tend to be systematically lower by $\approx 20\%$ than in the standard ΛCDM (dashed lines) at $z=0$.

file (Navarro, Frenk & White 1997) and concentrations are computed by the AHF code. We computed the mean $c(M)$ relation as in Kwan et al. (2012). In Figure 5 we show the mean c - M relation found. Halos at $z=0$ formed in the ArCDM cosmology tend to be $\approx 20\%$ less concentrated than in the standard ΛCDM , for most of the mass range of our halos; at higher masses ($\sim 10^{14} h^{-1} M_\odot$) the situation is not that clear. The ΛCDM simulations yields results which are consistent with those, for example, of Duffy et al. (2008). A simple fit to the ΛCDM results at $z=0$ yields $c(M) = 5.2(M_{\text{vir}}/M_{14})^{-0.81}$, with $M_{14} = 10^{14} h^{-1} M_\odot$, while for the ArCDM halos the coefficient in former expression is 4.7 with essentially the same slope. At higher redshifts the slope tends to flatten, as found in other works.

Substructure. The impact of the radiation term also is noticed in the number of subhaloes detected by the AHF CODE. In the ΛCDM model the number of subhaloes for the six most massive halos are $\{94, 47, 64, 36, 54, 51\}$ while for the ArCDM we obtained $\{55, 35, 14, 18, 19, 24\}$; i.e. the inclusion of Ω_r tends to reduce the number of subhaloes. In a future work we explore the substructure differences in more detail.

4 SUMMARY AND FINAL COMMENTS

We have carried out two numerical experiments on the evolution of N -body dark matter cosmological simulation, one with the usual neglect of the radiation energy density Ω_r term in the Hubble function and one that includes it.

Different diagnostics, such as the power density spectrum $P(k)$, the halo mass function, and the c - M relation, were used to quantify differences that occur when neglecting the radiation energy density. Other tests, such as the subhalo velocity distribution function are not considered in this work, in one part due to lacking the mass resolution for an adequate treatment and on other due reduced space and nature of this *Letter*. Future works will address some of these topics.

Including the radiation term makes the structure formation of the universe to lag in time in comparison to when it is not included. This may be understood, at least in the linear regime, from recalling the growing mode D_+ (3). Including Ω_r makes the expansion rate larger, hence the perturbation growth is reduced. The same kind of effect occurs when, for instance, the Ω_Λ is included -for instance- in a Einstein-de Sitter universe; both terms Ω_Λ and Ω_r enter the Hubble function (2) with the same sign. This enhancement of the Hubble drag in comparison with the standard Λ CDM treatment has also the effect of reducing in average the concentration of the halos formed; see Figure 5.

We have shown that including the radiation energy term in a high redshift ($z = 500$) simulation leads to important differences in the structures of the universe than when not including it. The power spectrum density “deposited” at all scales tend to be lower by $\approx 50\%$ in the Λ rCDM cosmology than in the Λ CDM. Also, the effect of the radiation density reflects itself in the mean concentration of halos, by lowering it $\approx 20\%$ at the current epoch.

All the results presented in this work point toward the necessity to include in simulations the Ω_r term in the Hubble function. This is particularly important for questions regarding the first structures formed in the universe and their evolution. Also it may bear importance in problems at galactic scale such as the “missing satellites” (Klypin et al. 1999, Bullock 2010) or the “too big to fail” (Boylan-Kolchin et al. 2011). Implications of the Ω_r on such problems are the subject of future works, as well as comparisons to observations.

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